

Relaxation mechanisms in three-dimensional metamaterial lens focusing

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We study the transient behaviors in focusing a point source with a metamaterial lens ($\epsilon = \mu = -1 + \delta$). We find the time evolutions of image formation are dictated by two relaxation mechanisms, namely, surface wave transport and absorption, determined, respectively, by $\text{Re}(\delta)$ and $\text{Im}(\delta)$. We show that image oscillations are inevitable in this three-dimensional configuration when $\text{Re}(\delta) \neq 0$ and establish relationships among the relaxation time, the resolution enhancement, and δ . © 2005 Optical Society of America

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Using a metamaterial slab (with $\epsilon = \mu = -1 + \delta$) as a lens to focus electromagnetic waves has attracted much attention recently. The idea originates from a pioneering analysis by Veselago¹ and from a recent analysis by Pendry, who showed that the resolution could exceed the diffraction limit.² Many theoretical studies of this effect have been made.^{3–14} It was gradually realized that a nonzero δ (either real^{11,12} or imaginary^{4–8,10}) is needed to avoid the field divergence problems⁹ that are encountered in the perfect lens situation (i.e., $\epsilon = \mu = -1$).² Most studies considered a two-dimensional model employing a line source^{3–12} and focused only on the stabilized image properties. The transient behaviors were neglected in most studies (except, for example, in Ref. 14). Although the sources are monochromatic, transient waves are still inevitable due to the switch-on process, and there are nontrivial consequences in such problems. For example, in a two-dimensional model with a purely imaginary δ , a finite-difference time-domain simulation showed that the image oscillates dramatically over time.⁵ This phenomenon was traced to a beating effect involving a vortexlike surface wave (SW), but the same effect was found not to be serious in the three-dimensional (3D) model that used a point source as long as δ was purely imaginary.¹⁵ In most practical situations, the working frequency may not be tuned exactly to where $\text{Re}(\epsilon)$ or $\text{Re}(\mu)$ is precisely -1 [i.e., $\text{Re}(\delta) \neq 0$ generally]. One purpose of this Letter is to show that serious beating effects are also inevitable even in the 3D model if $\text{Re}(\epsilon)$ or $\text{Re}(\mu)$ is not exactly -1 at the working frequency.

We apply a rigorous time-dependent Green's function approach¹⁵ to quantitatively study the transient behaviors in the 3D metamaterial lens focusing, which were not adequately explored previously.^{3–12} Our lens has $\epsilon = \mu = -1 + \delta$ at the working frequency where δ is generally complex. We adopted the following two approximations: First, we considered a monochromatic point source (working at a frequency ω_0)

with the simplest switch-on process: $\mathbf{J}(\mathbf{r}, t) = \hat{y}P_0\delta(\mathbf{r})\exp(-i\omega_0 t)\theta(t)$. Second, we assumed that the lens is made from a dispersive material, with $\epsilon(f) = \mu(f) = 1 - 200/[f(f + i\gamma)]$. With the lens placed at the xy plane bounded by $z = -d/2$ and $z = -3d/2$, we found an image formed in the xy plane at $z = -2d$. The time-dependent \mathbf{E} field can be written as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2\pi} \int d\omega \exp(-i\omega t) \mathbf{E}(\mathbf{r}, \omega) \frac{\omega}{\omega - \omega_0 + i\eta}, \quad (1)$$

where η is a positive infinitesimal number. Here, $\mathbf{E}(\mathbf{r}, \omega)$ is the field calculated at a frequency ω (it can be different from ω_0), obtainable following the method described in Ref. 13. For example, in the xz plane with $y=0$ we found for $z < -3d/2$ that

$$E_y{}^{3D}(x, z; \omega) = -\frac{i\mu_0 P_0}{8\pi} \int_0^\infty \frac{1}{k_{0z}} \exp(-ik_{0z}z) \left\{ T^{\text{TE}}(k_{\parallel}) \right. \\ \times [J_0(k_{\parallel}x) - J_2(k_{\parallel}x)] + \frac{k_{0z}^2}{k^2} T^{\text{TM}}(k_{\parallel}) \\ \left. \times [J_0(k_{\parallel}x) + J_2(k_{\parallel}x)] \right\} k_{\parallel} dk_{\parallel}, \quad (2)$$

where J_n are the Bessel functions, $k = \omega/c$, $k_{0z}^2 + k_{\parallel}^2 = (\omega/c)^2$, and T^{TE} and T^{TM} are the transmission coefficients for incident waves with $\mathbf{k} = (\mathbf{k}_{\parallel}, k_z)$ and TE or TM polarization. Integrations in Eqs. (1) and (2) were performed by an adaptive grid method¹⁵ that ensured fast convergence.

We now study how $\text{Re}(\delta)$ and $\text{Im}(\delta)$ affect the transient behavior. We note that $\epsilon(f_0) = \mu(f_0) \approx -1 + 0.4(f_0 - 10) + i0.2\gamma$ as $f_0 \rightarrow 10$, $\gamma \rightarrow 0$. Thus, near the perfect-lens condition, $\text{Re}(\delta)$ and $\text{Im}(\delta)$ can be tuned independently by changing f_0 and γ . Figure 1 shows the calculated field evolutions with different values of $\text{Re}(\delta)$,

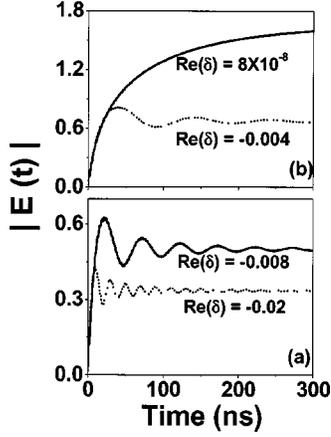


Fig. 1. Time evolution of $|E|$ (in units of $\mu_0 P_0$) at the image point for lenses with several values of $\text{Re}(\delta)$ at the working frequency. Here $\text{Im}(\delta)=0.0004$ and $d=10$ mm are fixed.

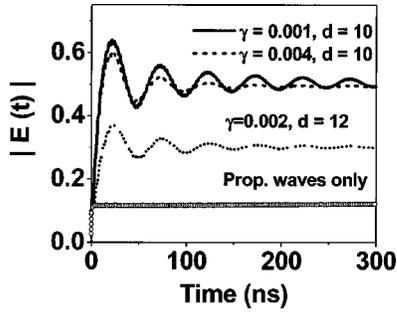


Fig. 2. Time evolution of $|E|$ (in units of $\mu_0 P_0$) measured at the image point for lenses with three values of γ (in units of gigahertz) and thicknesses d (in units of millimeters). $\text{Re}(\delta)$ is fixed as -0.004 . Open circles, results with only propagating components included for a lens with $\delta=-0.08 + 0.0002i$ and $d=10$ mm.

and Fig. 2 compares them with different values of $\text{Im}(\delta)$ and lens thicknesses d . It is clear that the field oscillation exists whenever $\text{Re}(\delta) \neq 0$, although all sources are monochromatic. In addition, the oscillations are clearly caused by the SWs, as the calculation including only propagating components shows no oscillation (open circles in Fig. 2). We found that $|E(t)|$ can be written approximately as

$$|E(t)| \approx E_{\text{av}}(t) + E_{\text{osc}} \sin(\bar{\omega}t + \phi) \exp(-\beta t), \quad (3)$$

where $E_{\text{av}}(t)$ is the averaged evolution part, E_{osc} is the oscillation strength, and $\bar{\omega}$ is the oscillation frequency. Several features can be summarized from Figs. 1 and 2:

- (1) $\bar{\omega}$ is determined solely by $\text{Re}(\delta)$ and has nothing to do with $\text{Im}(\delta)$ and d .
- (2) The role of $\text{Im}(\delta)$ is to damp out the oscillation. The oscillation will not stop if $\gamma=0$ as long as $\text{Re}(\delta) \neq 0$.
- (3) $E_{\text{av}}(t)$ is determined mainly by $\text{Re}(\delta)$. The evolution toward stabilization becomes slower when $\text{Re}(\delta) \rightarrow 0$.

To understand these features, in Fig. 3(a) we plotted the magnitude of the integrand in Eq. (1) as a function of frequency for $\eta=\gamma=0.001$ and $\text{Re}(\delta)=-0.04$ (achieved by setting $f_0=9.9$ GHz). In addition to the pole at f_0 , we found two other frequencies (denoted f_1 and f_2), which contribute appreciably to the integration in Eq. (1).¹⁶ We then plotted the SW spectrum^{17–19} in Fig. 3(b) for comparison. It is clear that the pole at f_1 corresponds to the band-edge SW state, which was previously determined to cause image oscillations in two dimension.¹⁵ The pole at f_2 corresponds to the frequency where $k_{\parallel} \rightarrow \infty$, achieved when $\epsilon=\mu \rightarrow -1$. To prevent field divergence problems⁹ the lens should have $\epsilon=\mu \neq -1$ at f_0 . However, the dispersive nature of the metamaterial implies that there must be another frequency at which $\epsilon=\mu \rightarrow -1$, which is the origin of the pole at f_2 .²⁰ The strength of the pole at f_2 is much stronger than that at f_1 , since the SWs for various k_{\parallel} are nearly degenerate at f_2 and also because f_2 is closer to f_0 [recall the factor $1/(\omega-\omega_0+i\eta)$ in Eq. (1)]. At a relatively long time, we expect that only the poles at f_0 and f_2 will dominate. Neglecting absorption, the total field approximately becomes

$$\begin{aligned} |E(t)| &= |E_0 \exp(i\omega_0 t) + E_2 \exp(i\omega_1 t)| \\ &\approx |E_0| + E_{\text{osc}} \sin(\bar{\omega}t + \phi), \end{aligned} \quad (4)$$

where $\bar{\omega}=\omega_2-\omega_0$, and $E_{\text{osc}}=|E_2|^2/|E_0|$ in the limit of $|E_2| \ll |E_0|$. As the pole at f_2 is determined solely by the intrinsic property (dispersion relation) of the lens material, a different lens thickness does not alter its position, as shown by the open circles in Fig. 3. This explains feature (1), that $\bar{\omega}$ does not depend on d . Since the pole at f_2 has no external energy input except at switch-on, a finite γ will eventually damp it out, and, in turn, the oscillation will die out. We thus have explained the second feature.

We emphasize that the present oscillation is different from that identified previously.^{5,15} First, the oscillation observed previously⁵ depends strongly on d , while the present oscillation does not. Second, the

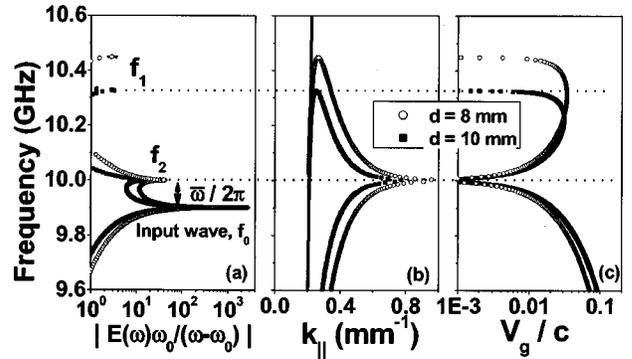


Fig. 3. (a) Calculated values of $|E(\omega)\omega_0/(\omega-\omega_0+i\eta)|$ as functions of frequency for several values of d . The SW spectra (TE and TM modes are degenerate) and the corresponding group velocity are shown in (b) and (c), respectively, for two values of d . In (c), only the branch with a lower group velocity (higher k_{\parallel}) is considered when $f > 10$ GHz.

previous oscillation⁵ was found when $\text{Re}(\delta)=0$, while the present one disappears in that limit [Fig. 1(b)]. The reason for such differences is that they correspond to beating effects that involve different modes, which exist in different situations.

As implied by relation (4), the averaged evolution part $E_{\text{av}}(t)$ is contributed mainly by the frequency components around f_0 . For this part, the field evolution can be viewed as the leaking of those transient waves with frequencies near f_0 . We believe that this process is conducted through lateral SW transport at a speed of group velocity $V_g = \partial\omega / \partial k_{\parallel}$. To test this assumption, in Fig. 3(c) we show the calculated V_g as a function of frequency. It is clear that V_g becomes smaller as $\text{Re}(\delta) \rightarrow 0$ (i.e., approaching the pole at f_2), which explains the third feature.

We now study the relaxation mechanism for $\text{Re}(\delta) = 0$. Because of the weak oscillation in this case, relaxation time t_R can be defined unambiguously. Let us define t_R as the time when $|E|$ reaches 95% of its stabilized value. We have calculated t_R and resolution enhancement R for lenses with various values of γ . Here R is defined as w_0/w , where w is the image peak width measured at its half-maximum and w_0 is the diffraction-limited value. R is plotted against γ in Fig. 4(a), which shows that a small γ indeed gives a better R , and R scales logarithmically with γ . We then plot t_R in Fig. 4(b) for different lenses, characterized by R achieved through such lenses. We find that t_R depends exponentially on R and that a perfect lens (with $\gamma=0$) requires infinite time to attain infinite resolution ($R \rightarrow \infty$). Combining Figs. 4(a) and 4(b), we note that $t_R \sim \gamma^\alpha$ and α is quite close to -1 . This is not a coincidence but reveals the underlying relaxation mechanism. Since $V_g \rightarrow 0$ as $\text{Re}(\delta)=0$ [Fig. 3(c)], the leaking through lateral SW transport is slow and the dominant mechanism is now absorption. Thus all physical quantities relax to their stabi-

lized values in a way that $E(t) \sim E(\infty) + [E(0) - E(\infty)]\exp(-\beta t)$, where $\beta \propto \gamma$. This explains power law relation $t_R \sim \gamma^{-1}$, which is independent of any specific criterion.²¹

In short, we have studied transient behaviors in 3D metamaterial lens focusing. We identified two relaxation mechanisms, found an intrinsic image oscillation, and established relationships among the physical quantities involved.

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20. We restrict our attention to the simple case $\epsilon = \mu$. There could be other choices for solving the field divergence problems, such as $\epsilon = -1 + \delta$, $\mu = 1/(-1 + \delta)$, $\epsilon = -1$, $\mu = -1 + \delta$, and $\epsilon = -1 + \delta$, $\mu = -1$. The time evolutions may be even more complex in those situations.
21. We have chosen other criteria to define t_R and have found that the functional relations do not change although the absolute values of t_R change.

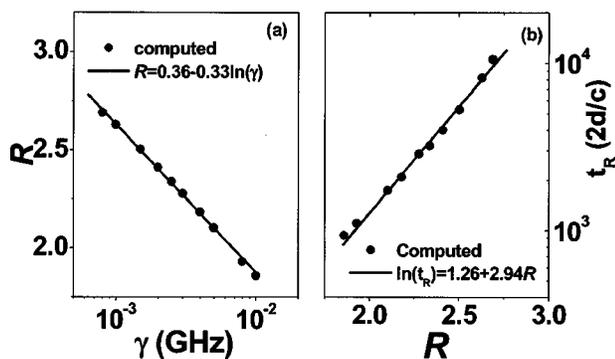


Fig. 4. (a) Calculated (symbols) resolution enhancement R as a function of γ , fitted by a logarithmic relation (solid line). (b) Relaxation time t_R in units of $2d/c$ (symbols) as a function of R achieved by a lens; solid line, best fit with $\ln(t_R) = a + bR$, where a and b are constants.